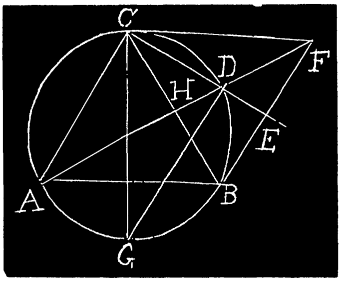
192. Proposed by ALFRED HUME, C. E., D. Ss., Professor of Mathematics, University of Mississippi, University, Miss.

Of all triangles with a common base and inscribed in the same circle, the isosceles is the maximum and has the maximum perimeter. Prove geometrically.

Solution by G. B. M. ZERR, A.M., Ph.D., The Temple College, Philadelphia, Pa., and JEANNETTE BROOKS, S. B., Chicago, Ill.

Let ABC be the isosceles triangle and ADB any other triangle, $AD>DB$. Produce AD to F, making $DF=DB$. Draw CF, BF, CD, DG. Then $\angle ADC=\angle FDE$; $\angle ADC$ is measured by $\frac{1}{2}(\text{arc}AC)=\frac{1}{2}(\text{arc}CDB)=\angle BDE$.

$\therefore\ \angle ADC=\angle BDE=\angle FDE$.



$\therefore\ CDE$ is perpendicular to BF at its mid-point. $\therefore\ CB=CF$.

Now $AC+CF>AF$. But $AF=AD+DB$, and $AC+CF=AC+CB$.

$\therefore\ AC+CB>AD+DB$. $\therefore\ AC+CB+AB>AD+DB+AB$.

$\dfrac{\triangle ADC}{\triangle BDC}=\dfrac{AC\times AD}{BC\times BD}$, but $AC=BC$ and $AD>BD$.

$\therefore\ \triangle ADC>\triangle BCD$. Take away the common triangle CHD and we get $\triangle AHC=\triangle BHD$.

$\therefore\ \triangle ABH+\triangle AHC>\triangle ABH+\triangle BHD$. $\therefore\ \triangle ABC>\triangle ABD$.

Also solved by *CLARENCE A. SHORT*, and *LON C. WALKER*.

CALCULUS.

158. Proposed by L. C. WALKER, A. M., Graduate Student, Leland Stanford Jr. University, Cal.

It is required to cut a hole a inches square, for a crank shaft, through the center of a grindstone b inches thick at the outer edge, c inches thick at the center, and d inches in diameter. How many cubic inches will have to be cut out?

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Ohio University, Athens, O., and G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.

The equation to a right circular cone, its base being the plane of x, y, and center the origin of coördinates, is $x^2+y^2=\dfrac{r^2}{h^2}(h-z)^2$; r, the radius of base, and h, the altitude. In this example, $h=\frac{1}{2}c$, and $r=\dfrac{cd}{2(c-b)}$. The required volume $=\int\int\int dxdydz$. The limits of z are $\dfrac{c}{2}\left(1-\dfrac{1}{r}\sqrt{(x^2+y^2)}\right)$, $-\dfrac{c}{2}\left(1-\dfrac{1}{r}\sqrt{(x^2+y^2)}\right)$; of y, $\frac{1}{2}a$, $-\frac{1}{2}a$; of x, $\frac{1}{2}a$, $-\frac{1}{2}a$.

$$\therefore \int\int\int dxdydz=c\int\int dxdy-\frac{c}{r}\int\int dxdy\sqrt{(x^2+y^2)}=ac\int dx$$